

QED radiative corrections to impact factors

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Abstract

We consider the radiative corrections to the impact factors of electron and photon. According to a generalized eikonal representation the e^+e^- scattering amplitude at high energies and fixed momentum transfers is proportional to the electron formfactor. But we show that this representation is violated due to the presence of non-planar diagrams. One loop correction to the photon impact factor for small virtualities of the exchanged photon is obtained using the known results for the cross section of the e^+e^- production at photon - nuclei interactions.

1 Introduction

It is well known (see [1]), that for the QED scattering amplitude of the process $a + b \rightarrow a' + b'$ in the Regge kinematics

$$\begin{aligned} A(p_A, a) + B(p_B, b) &\rightarrow A(p'_A, a') + B(p'_B, b'), \\ s = (p_A + p_B)^2 &\gg -t = -(p_A - p'_A)^2 \sim m^2, \end{aligned} \quad (1)$$

the simple representation

$$A(s, t) = \frac{is}{(2\pi)^2} \int \frac{d^2k \tau^A(k, r) \tau^B(k, r)}{[(k+r)^2 + \lambda^2][(k-r)^2 + \lambda^2]} (1 + O(\frac{t}{s})), \quad 4r^2 = -t < 0, \quad (2)$$

is valid in the first non-trivial order of the perturbation theory. Here λ is the photon mass and the impact factors (IF) τ describe the inner structure of colliding particles. The quantities r and k are two-dimensional vectors orthogonal to the momenta p_A, p_B of initial particles. For the electron $|\tau^e| = 4\pi\alpha\delta_{ij}$, where indices i, j describe its polarization states. The expression for IF of the photon on its mass shell can be presented in the form [1]:

$$\tau_{ij}^\gamma = 8\alpha^2 \int_0^1 dy \int_0^1 dx_+ dx_- \delta(x_+ + x_- - 1) (A_{ij} - B_{ij}), \quad (3)$$

with

$$\begin{aligned} A_{ij} = \frac{1}{4r^2 x_+^2 y(1-y) + m^2} & [8x_+^3 x_- y(1-y) r_i r_j \\ & - x_+^2 r^2 (1 - 8x_+ x_- (y - \frac{1}{2})^2) \delta_{ij}]; \end{aligned}$$

$$\begin{aligned}
B_{ij} &= \frac{1}{4Q^2 y(1-y) + m^2} [8x_+ x_- y(1-y) Q_i Q_j - Q^2 (1 - 8x_+ x_- (y - \frac{1}{2})^2) \delta_{ij}], \\
Q &= \frac{1}{2}(k+r) - x_+ r,
\end{aligned}$$

where i, j describe the photon polarization states.

From the point of view of the Regge theory the impact factors are proportional to the residue at $j = 1$ of the pole of the t -channel partial wave f_j describing the transition of two particles into the nonsense state of two virtual photons [1]. In the upper orders of the perturbation theory there appear the poles $f_j \sim 1/(j-1)^n$ which should be subtracted from τ providing one sums all logarithmic cotributions $\sim \log^n(s)$ with the use of the Bethe-Salpeter equation [1].

For $t = 0$ the impact factor is proportional to the integral from the total cross-section for the scattering of the virtual photon off the target. In the particular case, when the photon transverse momentum k tends to zero, the impact factors can be expressed in terms of the integrals from the total cross-section for the real photon

$$\tau = k^2 \int_{th}^{\infty} \frac{d s}{\pi} \frac{\sigma_{a\gamma}(s)}{s}, \quad (4)$$

which corresponds to the Weizsaekker-Williams approximation.

The motivation for our calculation of radiative corrections(RC) to impact factors is the high precision experiments performed on colliders where some physical quantities (for example, the BFKL Pomeron intercept) are measured [2]. In this case one should know IF of the virtual photon [2]. Generally IF describe the coupling of the particles with the Pomeron in QED or in QCD. For colliders with electron (positron) beams our results can be used to calculate the QED-part of cross-sections with a good accuracy. It turns out that the scattering amplitude corresponding to the diagrams with the two photon exchange is purely imaginary. In the case of the small angle e^+e^- scattering the amplitude for the diagrams with the multi-photon exchange has the eikonal representation

$$\begin{aligned}
A(s, t) &= A_0(s, t) e^{i\delta(t)}, \quad A_0(s, t) = 4\pi\alpha \frac{2}{st} \bar{u}(p'_1) \hat{p}_2 u(p_1) \bar{v}(p_2) \hat{p}_1 v(p'_2) = 4\pi\alpha \frac{2s}{t} N_1 N_2, \\
|N_i| &= 1, \delta(t) = -i\alpha \ln \frac{-t}{\lambda^2}.
\end{aligned} \quad (5)$$

Here we used the fact, that only longitudinal (nonsense) polarizations of the t -channel virtual photons are essential

$$\bar{u}(p'_1) \gamma_\mu u(p_1) \bar{v}(p_2) \gamma_\nu v(p'_2) G^{\mu\nu}(q), \quad G^{\mu\nu}(q) = \frac{1}{q^2} \frac{2p_2^\mu p_1^\nu}{s}. \quad (6)$$

RC to A_0 appear from the so-called "decorated boxes" (see Fig.1). These Feynman diagrams were assumed to lead to a generalized eikonal representation

$$A = A_0(s, t) [\Gamma_1(t)]^2 e^{i\delta(t)}, \quad (7)$$

where $\Gamma_1(t)$ is the Dirac form factor of electron

$$\begin{aligned} V^\mu(t) &= \gamma^\mu \Gamma_1(t) + \frac{\sigma^{\mu\nu} q_\nu}{2m} \Gamma_2(t), \quad q^2 = t, \\ \Gamma_1(t) &= 1 + \gamma \Gamma_1^{(2)}(t) + \dots, \quad \gamma = \frac{\alpha}{\pi}. \end{aligned} \quad (8)$$

Note, that one should include in $\delta(t)$ also corrections to the virtual photon Green function, leading in particular to the electric charge renormalization.

To begin with, in the next section we verify the generalized eikonal representation for the decorated boxes.

2 RC to IF of electron

Keeping in mind that the amplitude for the near forward scattering with the two photon exchange is pure imaginary (we omit the corrections of order m^2/s) we can calculate its s-channel discontinuity. There are RC to this discontinuity from the virtual photons and from the emission of the real photon in the intermediate state. The last contribution we will separate in two parts corresponding to the emission of soft and hard photons.

The virtual photon contribution contains the electron vertex function for the case when the initial and final electrons are on mass shell:

$$\begin{aligned} \Delta\tau_e^{virt} &= \frac{\alpha}{\pi} \tau_e^{(0)} [F_1^{(2)}(k^2) + F_1^{(2)}(k'^2)], \\ F_1^{(2)}(t) &= \ln \frac{m}{\lambda} (1 - \frac{1+a^2}{2a} \ln b) - 1 + \frac{1+2a^2}{4a} \ln b \\ &\quad - \frac{1+a^2}{2a} [-\frac{1}{4} \ln^2 b + \ln b \ln(1+b) - \int_1^b \frac{dx}{x} \ln(1+x)], \\ a &= \sqrt{1 - \frac{4m^2}{t}}, \quad b = \frac{a+1}{a-1}, \quad t < 0. \end{aligned} \quad (9)$$

The contribution from the emission of a soft photon have the classical form:

$$- \frac{\alpha}{4\pi^2} \left(\frac{p_1}{p_1 k_1} - \frac{p}{p k_1} \right) \left(\frac{p_1}{p_1 k_1} - \frac{p'_1}{p'_1 k_1} \right) \tau_e^{(0)} \times \frac{d^3 k_1}{\omega_1} |_{\omega_1 < \delta E}, \quad \delta E \ll E = \sqrt{s}/2, \quad (10)$$

where the momenta of initial and final electrons are p, p'_1 and the momentum of electron in the intermediate state is p_1 . Because the energies of these particles are approximately equal (and large in comparison with the electron mass) we can use the relations:

$$\begin{aligned} \frac{1}{2\pi} \int \frac{d^3 k_1}{\omega_1} \frac{m^2}{(p_i k_1)^2} &= 2L_e; \\ \frac{1}{2\pi} \int \frac{d^3 k_1}{\omega_1} \frac{p_1 p_2}{(p_1 k_1)(p_2 k_1)} &= \frac{1+a^2}{a} [L_e \ln b - \frac{1}{4} \ln^2 b] \end{aligned}$$

$$\begin{aligned}
& + \ln b \ln(1+b) - \int_1^b \frac{dx}{x} \ln(1+x)], \\
L_e &= \ln \Delta + \ln \frac{m}{\lambda}, \quad t = (p_1 - p_2)^2, \quad \Delta = \frac{\delta E}{E} \ll 1, \quad (11)
\end{aligned}$$

with the quantities a, b defined in (9).

At last we consider the hard photon emission. Its contribution to the imaginary part of the electron-electron scattering amplitude can be presented in the form

$$Im_s A(s, t) = -s \frac{\alpha^3}{2\pi^2} \int \frac{d^2 k}{k^2 k'^2} N_1 N_2 \frac{d^2 k_1 dx}{x(1-x)} I(x, k_1, k), \quad \Delta < x < 1, \quad (12)$$

where x is the energy fraction of the hard photon. We obtain

$$\begin{aligned}
I(x, k_1, k) &= \frac{1}{d_1 d_2} (-4m^2 + 2t_1 z) + \frac{1}{d_1 d'_1} (-4m^2 x^2 (1-x) + 2tz(1-x)) \\
&+ \frac{1}{d_2 d'_1} (-4m^2 + 2t_2 z) - 2z \frac{1}{d_1} - 4z \frac{1}{d_2} + \frac{8m^2}{d_2^2} - 2z \frac{1}{d'_1}, \quad z = 1 + (1-x)^2,
\end{aligned} \quad (13)$$

where

$$\begin{aligned}
d_1 &= (P - k_1)^2 - m^2 = -\frac{1}{x} [m^2 x^2 + \vec{k}_1^2], \\
d_2 &= (p_1 + k_1)^2 - m^2 = \frac{1}{x(1-x)} [m^2 x^2 + (x\vec{k} - \vec{k}_1)^2], \\
d'_1 &= (P' - k_1)^2 - m^2 = -\frac{1}{x} [m^2 x^2 + (x\vec{q} - \vec{k}_1)^2].
\end{aligned} \quad (14)$$

The subsequent integration is straightforward and gives the result:

$$\begin{aligned}
\Delta \tau_e^{hard} &= \tau_e^{(0)} \frac{\alpha}{\pi} \left[\ln \frac{1}{\Delta} (G(k^2) + G(k'^2) - G(t)) + G_1(k^2) + G_1(k'^2) - G_1(t) \right], \\
G(t) &= \frac{1+a^2}{2a} \ln b - 1; \quad G_1(t) = 1 - \frac{1+2a^2}{4a} \ln b.
\end{aligned} \quad (15)$$

The interference of two amplitudes with the photon emitted by two initial particles is small $\sim O(t/s)$. This fact is known in literature as the up-down cancellation. The contribution of the diagrams with the two photon exchange is pure imaginary and, consequently, does not interfere with the real Born amplitude. Adding together all contributions we obtain the final result for one loop RC to the electron IF

$$\Delta \tau_e = -\frac{\alpha}{\pi} \tau_e^{(0)} F_1^{(2)}(t), \quad \tau_e^{(0)} = 4\pi\alpha. \quad (16)$$

This result agrees with the generalized eikonal form of the small angle scattering amplitude. But in the upper orders the eikonal representation is violated, as it will be shown below.

3 Discussion

The above result for RC to the electron IF can be obtained in a simple way. For simplicity let us consider the case when the positron block consists only from the single fermion line whereas electron one contains the set of 4 Feynman graphs, describing the decorated box. We express the components of the exchanged photon momentum in terms of the squared invariant energies s_1, s_2 for electron and positron blocks

$$k = \alpha p_2 + \beta p_1 + k_\perp; \quad d^4 k = \frac{s}{2} d\alpha d\beta d^2 k_\perp = \frac{1}{2s} ds_1 ds_2 d^2 k_\perp;$$

$$s_1 = (k - p_1)^2 = s\alpha - \vec{k}^2; \quad s_2 = (k + p_2)^2 = s\beta - \vec{k}^2,$$

where the Sudakov parametrization is used. Performing the s_2 -integration by residue from the propagator of the intermediate positron (it takes into account also the diagram with the crossed photon lines), we obtain the following expression for the total RC

$$- \frac{8i\alpha^2}{st} \int \frac{d^2 \vec{k} \, \vec{q}^2}{(\vec{k}^2 + \lambda^2)((\vec{q} - \vec{k})^2 + \lambda^2)} \bar{v}(p_2) \hat{p}_1 v(p'_2) \int_C ds_1 p_2^\mu p_2^\nu \bar{u}(p'_1) A_{\mu\nu} u(p_1), \quad (17)$$

where $\bar{u}(p'_1) A_{\mu\nu} u(p_1)$ is the Compton scattering amplitude, corresponding to the Feynman diagrams having only s -channel singularities and the contour C is situated above these singularities. The amplitude has the pole at $s_1 = m^2$ which corresponds to the electron intermediate state and the right hand cut starting from $s_1 = (m + \lambda)^2$, which corresponds to one electron and one photon intermediate state.

Using the Sudakov representation for the photon momentum k we can present p_2^μ in the form

$$p_2^\mu = \frac{1}{\alpha} (k - k_\perp - \beta p_1) \approx - \frac{s}{s_1 + \vec{k}^2} (k - k_\perp)^\mu. \quad (18)$$

Let us consider the product of two terms in right side part of eq.(17) with the Compton amplitude. The contribution of the first term $\sim k_\perp$ is zero:

$$|\vec{k}| s p_2^\nu \int_C \frac{ds_1 k_\perp^\mu}{(s_1 + \vec{k}^2) |\vec{k}|} \bar{u}(p'_1) A_{\mu\nu}(s_1, k, k') u(p_1) = 0. \quad (19)$$

This conclusion follows from the convergence of the integral over the large circle in the s_1 plane and the absence of the left cut. The second property is valid for the planar Feynman graphs. The convergence of the integral follows from the fact that only physical transverse polarizations of the virtual photon with the momentum k contribute and the quantity $e^\mu p_2^\nu A_{\mu\nu}$, $\vec{e} = \vec{k}_\perp / |\vec{k}|$ behaves at large s_1 as m^2/s_1 .

The cancellation of contributions from the pole and the cut for planar graphs is the base for sum rules which relate cross sections of various production amplitudes

in the Weizsaekker-Williams approximation and the slope of Dirac formfactor at zero momentum transfer(see [9]).

Applying the Ward identity to the first term k in (17) we obtain:

$$p_2^\mu p_2^\nu \bar{u}(p_1') A_{\mu\nu}(s_1) u(p_1) = -\frac{se^2}{s_1} p_2^\mu \bar{u}(p_1') \Gamma^\mu(q) u(p_1), \quad s_1 \gg m^2. \quad (20)$$

Now the large circle integral gives the generalized eikonal result, which means in particular, that for arbitrary t the total contribution of the various intermediate states is not zero.

We see that RC to IF of electron contain infrared divergences, which are cancelled in the cross-section with the contribution of the inelastic process – the photon emission.

RC to the photon IF do not contain infrared singularities, but unfortunately we do not know the closed expression for them. We can estimate only their size at small virtualities of exchanged photon \vec{k}^2 and $q = 0$. This value can be extracted from the results of paper [3], where the one-loop correction to the cross section of pair production by photon on the coulomb field of nuclei was calculated:

$$\sum_{i=1}^{i=2} [\tau + \Delta\tau]_{ii}^\gamma(k, 0) = \frac{28\vec{k}^2\alpha^2}{9m^2} [1 + \delta_p], \quad \vec{k}^2 \ll m^2, \\ \delta_p = \frac{\alpha}{\pi} \frac{9}{14} \left(\frac{1128}{35} \zeta(3) - \frac{6971}{210} \right) = 0.009. \quad (21)$$

The contribution of the Pauli form factor in the lowest order of PT is suppressed by the factor $O(t/s)$, but it survives in higher orders of PT. Due to the known property of Dirac form factor $F_1(0) = 0$ we obtain (in an agreement with the results of paper[5]) the vanishing of RC for the case $t = 0$.

The generalized eikonal (GE) hypothesis is violated in the 2-loop approximation to IF [6]). This fact can be verified for the case $t = 0$. If the GE hypothesis would work, the complete compensation of contributions to the electron IF from different processes would take place. We show now that such compensation is absent on the 2-loop level. Let us consider the high-energy small-angle electron-positron scattering amplitude corresponding to the "decorated" three-loop diagrams with the two-photon exchange in the scattering channel. It has the imaginary part only in the s -channel (the diagrams with crossed photons in t -channel give the same contribution). We present the amplitude in the form

$$A(s, t) = \int \frac{d^4k}{(2\pi)^4} \frac{J_{\mu\nu}^{(A)} J_{\mu_1\nu_1}^{(B)}}{k^2(q-k)^2} g^{\mu\mu_1} g^{\nu\nu_1}. \quad (22)$$

Using the above simplification for the Green functions of the exchanged photons

$$g^{\mu\mu_1} \approx \frac{2}{s} p_A^{\mu_1} p_B^\mu; \quad g^{\nu\nu_1} \approx \frac{2}{s} p_A^{\nu_1} p_B^\nu,$$

and the Sudakov parametrization of the exchanged photon momentum in terms of the squared invariant masses $s_1 = s\alpha$ and $s_2 = -s\beta$ of the particles moving along directions of the initial particles p_A and p_B respectively, we arrive to the impact picture of the scattering amplitude (1) with:

$$\tau^A = \int_C \frac{ds_1}{2\pi} \frac{1}{s^2} J_{\mu\nu}^{(A)} p_B^\mu p_B^\nu; \quad \tau^B = \int_C \frac{ds_2}{2\pi} \frac{1}{s^2} J_{\mu\nu}^{(B)} p_A^\mu p_A^\nu, \quad (23)$$

where the quantities $(1/s^2)J_{\mu\nu}^{(i)}p_j^\mu p_j^\nu$, do not depend on s in the limit of large s . The integration contour C is displaced in correspondence with the Feynman prescription between the right and left hand side singularities of the amplitude.

As above τ^A has a pole corresponding to a single particle intermediate state, the right and left cuts, corresponding to the intermediate states with 2 and 3 particles.

For planar diagrams the left cut in s_1 -plane is absent. First time it appears at the two-loop level due to the presence of non-planar diagrams. The most important contribution from non-planar diagrams corresponds to the e^+e^- pair production by two virtual photons. The corresponding impact factor contains the divergency in s_1 related to the presence of two-photon intermediate states in the crossing channel. For the case of $t = 0$ it was calculated in ref [10]). We write down it here only in the Weizsaekker-Williams approximation, where it has the form of the sum rule for the Borsellino formulae describing the energy dependence of the total cross-section for the e^+e^- pair production in the electron-photon collisions:

$$\tau = k^2 \int_{th}^s \frac{ds_1}{\pi} \frac{\sigma(s_1)}{s_1} = a \ln^2(s/m^2) + b \ln(s/m^2) + c. \quad (24)$$

As it was discussed above, the logarithmic dependence on the upper limit in the integral over s_1 should be subtracted in a self-consistent way to avoid the double counting, because the logarithmic contributions are summed by the Bethe-Salpeter equation for the Pomeron in QED. It results

$$\tau(k, 0) = \frac{\alpha^3 k^2}{\pi m^2} \left(\frac{418}{27} - \frac{13}{2} \zeta(2) \right), \quad (25)$$

for electron closed loop and

$$\tau(k, 0) = \frac{\alpha^3 k^2}{\pi M^2} \left(\frac{3011}{324} - \frac{28}{9} \zeta(2) - \frac{107}{9} \ln \left(\frac{M}{m} \right) \right), \quad (26)$$

for muon closed loop. Here m, M are masses of electron and muon.

For the case of the diagrams without closed fermionic loops the impact factor appears from the interference of the Bethe-Heitler amplitudes of the pair production by virtual photon on electron due to the identity of electrons in the final state. It corresponds to the 3-fermion intermediate state in the u -channel of the Compton scattering amplitude. It was calculated in [7]:

$$\begin{aligned} \tau_{(k,0)}^{(l)} &\approx \frac{k^2}{m^2} \frac{\alpha^3}{\pi} \left[\frac{221}{315} + \frac{41549}{6300} \xi(2) - \frac{216}{105} \xi(3) - \frac{792}{105} \xi(2) \ln 2 \right] \\ &\approx \frac{k^2}{m^2} \frac{\alpha^3}{\pi} (-3.57). \end{aligned} \quad (27)$$

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